Name of Course	: **CBCS B.A. (Prog)**
Unique Paper Code	: **62351101_OC**
Name of Paper	: **Calculus**
Part	: **I**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. $$f(x) = \begin{cases} \frac{x(e^{1/x} - e^{-1/x})}{e^{1/x} + e^{-1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that f(x) is continuous but not derivable at $x = 0$

and also for the function

$$y = \frac{sin^{-1}x}{\sqrt{1-x^2}},$$

show that

$$(1 - x^2)y_{n+2} - (2n+3)x\, y_{n+1} - (n+1)^2 y_n = 0.$$

2. Let $$f(x) = \begin{cases} \frac{1}{2} - x, & if\ 0 < x < \frac{1}{2} \\ \frac{3}{2} - x, & if\ \frac{1}{2} < x < 1 \end{cases}$$

$$f(0) = 0, f\left(\frac{1}{2}\right) = \frac{1}{2}, f(1) = 1\,.$$

Show that f(x) is discontinuous at $x = \frac{1}{2}, 1$ and also if $z = sec^{-1}\left(\frac{x^2+y^2}{x+y}\right)$

show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2cotz.$$

and also if $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$

show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

3. Show that the tangents to the curve

$$x^3 + y^3 = 3axy$$

at the points where it meets the parabola $y^2 = ax$ are parallel to the axis of y

and find the asymptotes of the curve

$$4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0.$$

4. Find the position and nature of the double points on the curve

$$y(y-6) = x^2(x-2)^3 - 9$$

and trace the curve $ay^2 = (x-a)(x-5a)^2$.

5. Verify the Rolle's Theorem for the function $f(x) = x(x-2)^3$ in [0,2]

and show that for $x > 0$

$$\frac{tanx}{x} > \frac{x}{sinx}, 0 < x < \pi/2.$$

6. Assuming the possibility of expansion prove that

$$\log secx = \frac{x^2}{2} + \frac{x^4}{12} + \cdots$$

and also verify Lagrange's Mean Value Theorem for

$f(x) = x^2 - 3x - 1$ in [1,3].